Be Ahead with Sustained Excellence

CET - 2010

Question paper with Solutions

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CET - 2010 Mathematics (Version – D1)

60 Questions
60 Marks
Duration: 70 Minutes

Question paper with Solutions

1. If a, –a, b are the roots of $x^3 - 5x^2 - x + 5 = 0$, then b is a root of ________

   (1) $x^2 - 3x - 10 = 0$ (2) $x^2 + 5x - 30 = 0$ (3) $x^2 + 3x - 20 = 0$ (4) $x^2 - 5x + 10 = 0$

   Ans (1)

   The sum of all the roots $a - a + b = 5 \Rightarrow b = 5$

   $x = 5$ satisfies the equation $x^2 - 3x - 10 = 0$

2. In the binomial expansion of $(1 + x)^{15}$, the coefficients of x^r and x^{r+3} are equal. Then r is ________

   (1) 4 (2) 6 (3) 8 (4) 7

   Ans (2)

   $T_{v+1} = {}^{15}C_r x^r$

   By data, the coefficient of x^r = coefficient of x^{r+3}

   This implies ${}^{15}C_r = {}^{15}C_{r+3}$

   $\Rightarrow {}^{15}C_{15-r} = {}^{15}C_{r+3} \quad \because {}^nC_r = {}^nC_{n-r}$

   $\therefore 15 - r = r + 3 \Rightarrow r = 6$

3. The n^{th} term of the series 1 + 3 + 7 + 13 + 21 + … is 9901. The value of n is ________

   (1) 900 (2) 99 (3) 100 (4) 90

   Ans (3)

   $S_n = 1 + 3 + 7 + 13 + 21 + \ldots + T_n$

   $S_n = \quad 1 + 3 + 7 + 13 + \ldots + T_{n-1} + T_n$

   $O = 1 + 2 + 4 + 6 + 8 + \ldots (T_n - T_{n-1}) - T_n$

   $T_n = 1 + [2 + 4 + 6 + 8 + \ldots \text{ to } (n - 1) \text{ terms}]$

   $$9901 = 1 + \frac{(n-1)}{2}[2a + (n-2)\,d]$$

   $$\Rightarrow 9900 = \frac{(n-1)}{2}[4 + (n-2)\cdot 2]$$

   $9900 = n\,(n-1)$

   $= 100 \times (100 - 1)$

   $= 100 \times 99$

   $\therefore n = 100$

4. If $\frac{1}{(3-5x)(2+3x)} = \frac{A}{3-5x} + \frac{B}{2+3x}$, then A : B is ________

(1) 3 : 5 (2) 3 : 2 (3) 2 : 3 (4) 5 : 3

Ans (4)

$1 = A(2+3x) + B(3-5x)$

By putting $x = \frac{3}{5}$, we get $1 = A\left(2+\frac{9}{5}\right)$ $\therefore A = \frac{5}{19}$

By putting $x = -\frac{2}{3}$, we get $1 = B\left(3+\frac{10}{3}\right)$ $\therefore B = \frac{3}{19}$

$\therefore$ A : B = 5 : 3

5. Which of the following is NOT true?

(1) $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$

(2) $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

(3) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$ is a tautology

(4) $\{(p \rightarrow q) \wedge (q \wedge r)\} \rightarrow (p \rightarrow r)$ is a tautology

Ans (3)

We know that $\sim(p \rightarrow q) \equiv p \wedge \sim q$

$\therefore \sim(p \rightarrow q)$ and $(p \rightarrow q)$ are not logically equivalent.

Hence $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$ is not a tautology.

6. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $A^2 + xA + yI = 0$ for (x, y) = ________

(1) (4, –1) (2) (1, 3) (3) (–4, 1) (4) (–1, 3)

Ans (3)

$A^2 + x \cdot A + yI = 0$

$\Rightarrow \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + x\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3x & 2x \\ x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3x+y & 2x \\ x & x+y \end{bmatrix} = \begin{bmatrix} -11 & -8 \\ -4 & -3 \end{bmatrix}$

$\therefore x = -4$ and $x + y = -3$

$\therefore y = 1$

7. The constant term of the polynomial $\begin{vmatrix} x+3 & x & x+2 \\ x & x+1 & x-1 \\ x+2 & 2x & 3x+1 \end{vmatrix}$ is ________

(1) –1 (2) 1 (3) 0 (4) 2

Ans (1)

For $x = 0$ we have $\begin{vmatrix} 3 & 0 & 2 \\ 0 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix} = 3 - 4 = -1$

8. If $\vec{a}, \vec{b}$ and $\vec{c}$ are nonzero coplanar vectors, then $[2\vec{a}-\vec{b} \;\; 3\vec{b}-\vec{c} \;\; 4\vec{c}-\vec{a}] =$ ________

(1) 27 (2) 9 (3) 25 (4) 0

Ans (4)

$[2\vec{a}-\vec{b} \;\; 3\vec{b}-\vec{c} \;\; 4\vec{c}-\vec{a}]$

$= (2\vec{a}-\vec{b})\cdot\{(3\vec{b}-\vec{c})\times(4\vec{c}-\vec{a})\}$

$= (2\vec{a}-\vec{b})\cdot\{12(\vec{b}\times\vec{c}) - 3(\vec{b}\times\vec{a}) + (\vec{c}\times\vec{a})\} = 24\vec{a}\cdot(\vec{b}\times\vec{c}) - 0 + 0 - 0 + 0 - \vec{b}\cdot(\vec{c}\times\vec{a}) = 0 - 0 = 0$

$\because \vec{a}\cdot(\vec{b}\times\vec{c}) = 0$

Aliter: Since the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, $2\vec{a}-\vec{b}$, $3\vec{b}-\vec{c}$ and $4\vec{c}-\vec{a}$ are also coplanar.

Hence their scalar triple product is zero.

9. A space vector makes the angles 150° and 60° with the positive direction of X and Y axes. The angle made by the vector with the positive direction of Z-axis is ________

(1) 180° (2) 120° (3) 90° (4) 60°

Ans (3)

$\alpha = 150°$, $\beta = 60°$, to find γ.

We have $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$\frac{3}{4} + \frac{1}{4} + \cos^2\alpha = 1$

$\therefore \cos^2\gamma = 0$

$\therefore \gamma = 90°$

10. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, then $3\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} =$ ________

(1) −3 (2) 3 (3) −1 (4) 1

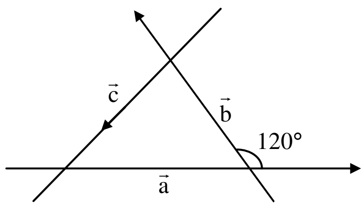
Ans (1)

Since $\vec{a}, \vec{b}, \vec{c}$ are unit vectors and $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ the vectors form the sides of an equilateral triangle.

Given expression $= 3\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$

$= 3\cdot1\cdot1\cdot\cos120° + 2\cdot1\cdot1\cdot\cos120° + 1\cdot1\cdot\cos120°$

$= -\frac{3}{2} - \frac{2}{2} - \frac{1}{2} = -3$



11. The greatest value of x satisfying 21 ≡ 385 (mod x) and 587 ≡ 167 (mod x) is ________

(1) 28 (2) 56 (3) 156 (4) 32

Ans (1)

$x \mid 21-385$ and $x \mid 587-167 \Rightarrow x \mid -364$ and $x \mid 420$

$x \mid 28 \times -13$ and $x \mid 28 \times 15$

The greatest value of x is 28.

12. The number $(49^2 - 4)(49^3 - 49)$ is divisible by ________

(1) 6 ! (2) 5 ! (3) 7 ! (4) 9 !

Ans (2)

Given expression $= (49^2 - 2^2) \cdot 49\,(49^2 - 1)$

$= 51 \cdot 47 \cdot 49 \cdot 50 \cdot 48$

$= 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47$ is the product of five consecutive integers.

Hence it is divisible by 5!.

13. The least positive integer x satisfying $2^{2010} \equiv 3x \pmod 5$ is ________

(1) 1 (2) 2 (3) 3 (4) 4

Ans (3)

We have $2^2 \equiv -1 \pmod 5$

$\Rightarrow (2^2)^{1005} \equiv (-1)^{1005} \pmod 5$

$2^{2010} \equiv -1 \pmod 5 \equiv 4 \pmod 5$

By data $2^{2010} \equiv 3x \pmod 5$

$\therefore 3x \equiv 4 \pmod 5$

$\therefore x = 3.$

14. If A and B are two square matrices of the same order such that AB = B and BA = A, then $A^2 + B^2$ is always equal to________

(1) 2AB (2) 2BA (3) I (4) A + B

Ans (4)

By data AB = B and BA = A

Consider $A^2 + B^2 = A \cdot A + B \cdot B$

$= A \cdot BA + B \cdot AB$

$= AB \cdot A + BA \cdot B$

$= BA + AB$

$= A + B$

15. If A is a 3×3 non-singular matrix and if $|A| = 3$ then, $|(2A)^{-1}| =$ ________

(1) $\frac{1}{3}$ (2) $\frac{1}{24}$ (3) 24 (4) 3

Ans (2)

We have $(2A) \cdot (2A)^{-1} = I$

$\Rightarrow |2A|\,|(2A)^{-1}| = |I|$

$2^3 |A| \cdot |(2A)^{-1}| = |I|$

$2^3 \cdot 3 \cdot |(2A)^{-1}| = 1$

$\therefore\ |(2A)^{-1}| = \dfrac{1}{24}$

16. Let R be an equivalence relation defined on a set containing 6 elements. The minimum number of ordered pairs that R should contain is ________

(1) 64 (2) 36 (3) 12 (4) 6

Ans (4)

Since, R is an equivalence relation defined on a set having 6 elements, R should be reflexive symmetric and transitive.

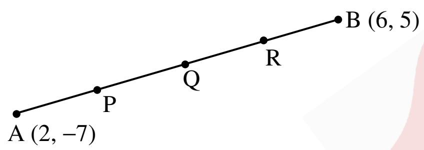
For R to be reflexive, $(a, a) \in R$ for all a.

Thus R must have atleast 6 ordered pairs.

17. The line joining A(2, −7) and B(6, 5) is divided into 4 equal parts by the points P, Q and R such that AQ = RP = QB. The midpoint of PR is ________

(1) (4, −1) (2) (8, −2) (3) (4, 12) (4) (−8, 1)

Ans (1)



Q is the mid-point of PR and also the mid point of AB.

Thus, the coordinates of $Q \equiv \left(\dfrac{2+6}{2}, \dfrac{-7+5}{2}\right) = (4, -1)$

18. Let $P \equiv (-1, 0)$, $Q \equiv (0, 0)$ and $R \equiv (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is ________

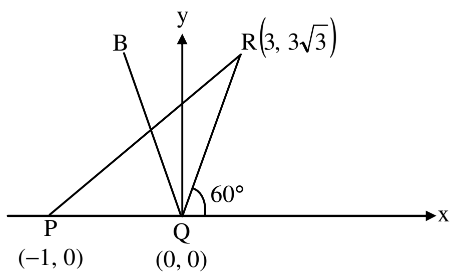
(1) $x + \sqrt{3}y = 0$ (2) $\sqrt{3}x + y = 0$ (3) $x - \sqrt{3}y = 0$ (4) $\sqrt{3}x - y = 0$

Ans (2)

The bisector QB of the angle PQR makes an angle of 120° with positive X-axis.

Its slope is $\tan 120° = -\sqrt{3}$.

Hence its equation is $y = -\sqrt{3}\ x$



19. If m is the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$, then $(h + bm)^2 =$ ________

(1) $h^2 + ab$ (2) $h^2 - ab$ (3) $(a + b)^2$ (4) $(a - b)^2$

Ans (2)

Since one of the lines has slope m, its equation is y = mx.

By putting y = mx in $ax^2 + 2hxy + by^2 = 0$ we get

$ax^2 + 2hmx^2 + bm^2x^2 = 0$

$\Rightarrow a + 2hm + bm^2 = 0$

$\Rightarrow ab + 2hmb + b^2m^2 = 0$ …(1)

Consider $(h + bm)^2$

$= h^2 + b^2m^2 + 2hbm$

$= h^2 - ab$ using (1)

20. cot 12° cot 102° + cot 102° cot 66° + cot 66° cot 12° = ________

(1) −1 (2) 2 (3) −2 (4) 1

Ans (4)

Let A = 12°, B = 102°, C = 66°

We find that A + B + C = 180°

$\Rightarrow \sum \cot A \cot B = 1$

21. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the positive direction of X, Y and Z-axes, then a FALSE statement in the following is ________

(1) $\sum \hat{i} \cdot (\hat{j} \times \hat{k}) = \vec{0}$ (2) $\sum \hat{i} \cdot (\hat{j} + \hat{k}) = \vec{0}$

(3) $\sum \hat{i} \times (\hat{j} + \hat{k}) = \vec{0}$ (4) $\sum \hat{i} \times (\hat{j} \times \hat{k}) = \vec{0}$

Ans (1)

$\sum i \cdot (j \times k) = \sum i \cdot i = \sum 1 = 3$

22. In P(X), the power set of a nonempty set X, a binary operation ∗ is defined by A ∗ B = A ∪ B ∀ A, B ∈ P(X). Under ∗, a TRUE statement is ________

(1) commutative law is not satisfied (2) associative law is not satisfied

(3) identity law is not satisfied (4) inverse law is not satisfied

Ans (4)

23. The inverse of 2010 in the group Q^+ of all positive rationals under the binary operation ∗ defined by $a * b = \frac{ab}{2010}$, $\forall a, b \in Q^+$, is ________

(1) 1 (2) 2010 (3) 2009 (4) 2011

Ans (2)

e = 2010 $\therefore e^{-1} = 2010$

24. If the three functions f(x), g(x) and h(x) are such that h(x) = f(x) · g(x) and f′(x) · g′(x) = c, where c is a constant, then $\frac{f''(x)}{f(x)} + \frac{g''(x)}{g(x)} + \frac{2c}{f(x)\cdot g(x)}$ is equal to ________

(1) $\frac{h''(x)}{h(x)}$ (2) $\frac{h(x)}{h'(x)}$ (3) $h'(x) \cdot h''(x)$ (4) $\frac{h(x)}{h''(x)}$

Ans (1)

Given function $= \frac{f''(x)\cdot g(x) + g''(x)\cdot f(x) + 2f'(x)\cdot g'(x)}{f(x)\cdot g(x)}$

Now h(x) = f(x) · g(x)

$h'(x) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$

$h''(x) = f(x)\cdot g''(x) + g'(x)\cdot f'(x) + g(x)\cdot f''(x) + f'(x)\cdot g'(x)$

$= f(x)\cdot g''(x) + g(x)\cdot f''(x) + 2f'(x)\cdot g'(x)$

$\therefore$ Given function $= \frac{h''(x)}{h(x)}$

25. The derivative of $e^{ax}\cos bx$ with respect to x is $re^{ax}\cos\left(bx + \tan^{-1}\frac{b}{a}\right)$. When a > 0, b > 0, the value of r is ________

(1) ab (2) a + b (3) $\sqrt{a^2+b^2}$ (4) $\frac{1}{\sqrt{ab}}$

Ans (3)

$y = e^{ax}\cos bx$

$\frac{dy}{dx} = e^{ax}\cdot(-b\sin bx) + \cos bx\cdot(ae^{ax})$

$= e^{ax}\{a\cos bx - b\sin bx\}$

$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\left\{\frac{a}{\sqrt{a^2+b^2}}\cos bx - \frac{b}{\sqrt{a^2+b^2}}\cdot\sin bx\right\}$

$= r\cdot e^{ax}\cdot\{\cos\theta\cdot\cos bx - \sin\theta\cdot\sin bx\}$

$= re^{ax}\cdot\{\cos(bx+\theta)\}$ where $r = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\frac{b}{a}$

26. The chord of the circle $x^2 + y^2 - 4x = 0$ which is bisected at (1, 0) is perpendicular to the line________

(1) x = 1 (2) y = 1 (3) y = x (4) x + y = 0

Ans (1)

Centre of the circle is C = (2, 0)

Let A = (1, 0)

AC is along x-axis.

The chord bisected at A is ⊥r to AC. ∴ The chord is ⊥r to x-axis at (1, 0)

Its equation is x = 1.

27. In ΔABC, if $a = 2$, $B = \tan^{-1}\frac{1}{2}$ and $C = \tan^{-1}\frac{1}{3}$, then $(A, b) =$

(1) $\left(\frac{3\pi}{4}, \frac{2\sqrt{2}}{\sqrt{5}}\right)$ (2) $\left(\frac{\pi}{4}, \frac{2}{\sqrt{5}}\right)$

(3) $\left(\frac{3\pi}{4}, \frac{2}{\sqrt{5}}\right)$ (4) $\left(\frac{\pi}{4}, \frac{2\sqrt{2}}{\sqrt{5}}\right)$

Ans (1)

$B + C = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

$\therefore A = \frac{3\pi}{4}$; $\tan B = \frac{1}{2} \Rightarrow \sin B = \frac{1}{\sqrt{5}}$

$$\frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow b = \frac{a \cdot \sin B}{\sin A} = \frac{2 \cdot \frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{5}}$$

28. The straight line $2x + 3y - k = 0$, $k > 0$ cuts the X- and Y-axes at A and B. The area of ΔOAB, where O is the origin, is 12 sq. units. The equation of the circle having AB as diameter is ________

(1) $x^2 + y^2 - 6x + 4y = 0$ (2) $x^2 + y^2 - 4x - 6y = 0$

(3) $x^2 + y^2 - 6x - 4y = 0$ (4) $x^2 + y^2 + 4x - 6y = 0$

Ans (3)

For the line $2x + 3y - k = 0$; $OA = a = \frac{k}{2}$ and $OB = b = \frac{k}{3}$

$\Delta AOB = 12 \Rightarrow \frac{1}{2}ab = 12 \Rightarrow \frac{1}{2}\cdot\frac{k}{2}\cdot\frac{k}{3} = 12 \Rightarrow k^2 = 144 \Rightarrow k = 12$

$\therefore OA = a = 6$ and $OB = b = 4$.

Equation of the circle with AB as diameter is $x^2 + y^2 - ax - by = 0$

$\Rightarrow x^2 + y^2 - 6x - 4y = 0$

29. Let $P(x, y)$ be the midpoint of the line joining (1, 0) to a point on the curve $y^2 = \begin{vmatrix} x+1 & x+2 \\ x+3 & x+5 \end{vmatrix}$. The locus of P is symmetrical about ________

(1) $x = 1$ (2) $y = 1$ (3) Y-axis (4) X-axis

Ans (4)

$y^2 = (x + 1)(x + 5) - (x + 3)(x + 2)$

$= (x^2 + 6x + 5) - (x^2 + 5x + 6)$

$y^2 = x - 1$

This is a parabola with vertex at (1, 0) and axis x-axis.

The locus of the mid-point of the vertex and any point on the parabola is also a parabola with the same axis i.e., x-axis.

30. The function f(x) = |x − 2| + x is ________

(1) continuous at x = 2 but not at x = 0

(2) continuous at both x = 2 and x = 0

(3) differentiable at both x = 2 and x = 0

(4) differentiable at x = 2 but not at x = 0

Ans (2)

f(x) = |x − a| is continuous at x = a but not differentiable at x = a.

31. (sin θ + cos θ) (tan θ + cot θ) = ________

(1) sec θ + cosec θ

(2) sec θ . cosec θ

(3) sin θ . cos θ

(4) 1

Ans (1)

$$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = (\sin\theta + \cos\theta)\cdot\left(\frac{1}{\sin\theta\cos\theta}\right)$$

$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \sec\theta + \operatorname{cosec}\theta$$

32. The sides of a triangle are $6+\sqrt{12}$, $\sqrt{48}$ and $\sqrt{24}$. The tangent of the smallest angle of the triangle is ________

(1) $\sqrt{3}$

(2) 1

(3) $\frac{1}{\sqrt{3}}$

(4) $\sqrt{2}-1$

Ans (3)

$a = 6+\sqrt{12}$, $b=\sqrt{48}$, $c=\sqrt{24}$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{36+12+12\sqrt{12}+48-24}{2\left(6+\sqrt{12}\right)\cdot\sqrt{48}}$$

$$= \frac{72+24\sqrt{3}}{2\cdot\left(6+2\sqrt{3}\right)\cdot 4\sqrt{3}} = \frac{24\left(3+\sqrt{3}\right)}{2\cdot 3\left(3+\sqrt{3}\right)\cdot 4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$\therefore$ C = 30° and $\tan C = \frac{1}{\sqrt{3}}$

33. A simple graph contains 24 edges. Degree of each vertex is 3. The number of vertices is ________

(1) 21

(2) 16

(3) 8

(4) 12

Ans (2)

Sum of the degrees of all vertices = 2 (number of edges)

3n = 2(24)

⇒ n = 16 = number of vertices.

34. $\lim_{n\to\infty}\left\{n\sin\frac{2\pi}{3n}\cdot\cos\frac{2\pi}{3n}\right\} =$ ________

(1) 1

(2) $\frac{\pi}{3}$

(3) $\frac{\pi}{6}$

(4) $\frac{2\pi}{3}$

Ans (4)

$$\lim_{x\to\infty} n\cdot\left\{\frac{\sin\frac{2\pi}{3n}}{\frac{2\pi}{3n}}\right\}\cdot\frac{2\pi}{3n}\cdot\cos\frac{2\pi}{3n} = \lim_{x\to\infty}\left\{\frac{\sin\frac{2\pi}{3n}}{\frac{2\pi}{3n}}\right\}\cdot\frac{2\pi}{3}\cdot\cos\frac{2\pi}{3n}$$

$$= (1)\cdot\frac{2\pi}{3}\cdot 1 = \frac{2\pi}{3}$$

35. The function f(x) = [x], where [x] denotes the greatest integer not greater than x, is ________

(1) continuous for all real values of x

(2) continuous only at rational values of x

(3) continuous for all non-integral values of x

(4) continuous only at positive integral values of x

Ans (3)

f(x) = [x] is a step function which is not continuous at any integral values of x.

If x = n = any integer then, infact $\lim_{x\to n^-} f(x) = n-1$ and $\lim_{x\to n^+} f(x) = n$

$\therefore \lim_{x\to n} f(x)$ also does not exist.

36. If $a > b > 0$, $\sec^{-1}\left(\frac{a+b}{a-b}\right) = 2\sin^{-1} x$, then x = ________

(1) $-\sqrt{\frac{a}{a+b}}$

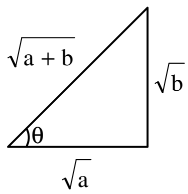
(2) $\sqrt{\frac{a}{a+b}}$

(3) $-\sqrt{\frac{b}{a+b}}$

(4) $\sqrt{\frac{b}{a+b}}$

Ans (4)

$$\sec^{-1}\left(\frac{a+b}{a-b}\right) = \sec^{-1}\left\{\frac{1+\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^2}{1-\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^2}\right\} = 2\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right)$$



$$\therefore \sin^{-1} x = \tan^{-1}\left(\sqrt{\frac{b}{a}}\right) = \theta \quad \therefore \tan\theta = \sqrt{\frac{b}{a}}$$

$$x = \sin\theta = \frac{\sqrt{b}}{\sqrt{a+b}}$$

37. If $x \neq n\pi$, $x \neq (2n+1)\frac{\pi}{2}$, $n \in Z$, then $\frac{\sin^{-1}(\cos x)+\cos^{-1}(\sin x)}{\tan^{-1}(\cot x)+\cot^{-1}(\tan x)} =$ ________

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{3}$

(3) $\frac{\pi}{2}$

(4) $\frac{\pi}{6}$

Ans (–)

None of the given options is correct

The correct answer is = 1

$$\text{Given expression } = \frac{\frac{\pi}{2}-x+\frac{\pi}{2}-x}{\frac{\pi}{2}-x+\frac{\pi}{2}-x} = 1$$

38. The general solution of $1 + \sin^2 x = 3\sin x \cdot \cos x,\ \tan x \neq \frac{1}{2}$ is ________

(1) $n\pi - \frac{\pi}{4},\ n \in Z$ (2) $n\pi + \frac{\pi}{4},\ n \in Z$

(3) $2n\pi + \frac{\pi}{4},\ n \in Z$ (4) $2n\pi - \frac{\pi}{4},\ n \in Z$

Ans (2)

$1 + \sin^2 x = 3 \sin x \cdot \cos x$

Dividing throughout by $\cos^2 x$

$\Rightarrow \sec^2 x + \tan^2 x = 3 \tan x$

$\Rightarrow 2 \tan^2 x - 3 \tan x + 1 = 0$

$\Rightarrow (2 \tan x - 1) \cdot (\tan x - 1) = 0$

Here $\tan x \neq \frac{1}{2}$ $\therefore \tan x = 1$

$\Rightarrow x = n\pi + \frac{\pi}{4};\ \ n \in Z$

39. The least positive integer n, for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is positive, is ________

(1) 1 (2) 2 (3) 3 (4) 4

Ans (1)

$$G.E. = \frac{(1+i)^n}{(1-i)^n \cdot (1-i)^{-2}} = \left(\frac{1+i}{1-i}\right)^2 \cdot (1-i)^2 = i^n(-2i)\ \ s = -2 \cdot i^{n+1}$$

When $n = 1$, G.E. = 2 = real.

40. If $x + iy = \left(-1 + i\sqrt{3}\right)^{2010}$, then $x =$ ________

(1) 1 (2) −1 (3) -2^{2010} (4) 2^{2010}

Ans (4)

$$x + iy = \left(-1 + i\sqrt{3}\right)^{2010} = (2\omega)^{2010} = 2^{2010} \cdot \omega^{2010} = 2^{2010} \cdot (\omega^3)^{67} = 2^{2010}$$

41. The condition for the line $y = mx + c$ to be a normal to the parabola $y^2 = 4ax$ is ________

(1) $c = \frac{a}{m}$ (2) $c = 2am + am^3$ (3) $c = -2am - am^2$ (4) $c = -\frac{a}{m}$

Ans (3)

Book work

42. The eccentric angle of the point $\left(2, \sqrt{3}\right)$ lying on $\frac{x^2}{16} + \frac{y^2}{4} = 1$ is ________

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans (1)

Any point on the given ellipse can be taken as $(4\cos\theta, 2\sin\theta)$

$\Rightarrow (2, \sqrt{3}) = (4\cos\theta, 2\sin\theta)$

$\therefore 4\cos\theta = 2 \Rightarrow \cos\theta = \frac{1}{2} \quad \therefore \theta = \frac{\pi}{3}$

43. The distance of the focus of $x^2 - y^2 = 4$, from the directrix which is nearer to it, is

(1) $2\sqrt{2}$ (2) $\sqrt{2}$ (3) $4\sqrt{2}$ (4) $8\sqrt{2}$

Ans (2)

$SZ = ae - \frac{a}{e}$

For $x^2 - y^2 = 4 \Rightarrow a = 2, \; e = \sqrt{2}$

$\therefore SZ = 2\sqrt{2} - \frac{2}{\sqrt{2}} = \sqrt{2}$

44. If $\int f(x)\sin x \cdot \cos x\,dx = \frac{1}{2(b^2 - a^2)}\log f(x) + c$, where c is the constant of integration, then f(x) =

(1) $\frac{2}{ab\sin 2x}$ (2) $\frac{2}{(b^2 - a^2)\sin 2x}$ (3) $\frac{2}{ab\cos 2x}$ (4) $\frac{2}{(b^2 - a^2)\cos 2x}$

Ans (4)

Given $\int f(x)\sin x \cdot \cos x \cdot dx = \frac{1}{2(b^2 - a^2)}\log f(x) + c$

$$\Rightarrow \frac{d}{dx}\left[\frac{1}{2(b^2 - a^2)}\log f(x) + c\right] = f(x)\sin x \cdot \cos x$$

$$\Rightarrow \frac{1}{2(b^2 - a^2)}\frac{1}{f(x)}f'(x) = \frac{f(x)}{2}\cdot(2\sin x \cdot \cos x)$$

$\therefore f'(x) = [f(x)]^2 (b^2 - a^2)\sin 2x$

Let $f(x) = y \Rightarrow f'(x) = \frac{dy}{dx}$

$\therefore \frac{dy}{dx} = y^2(b^2 - a^2)\sin 2x$

$\int \frac{dy}{y^2} = (b^2 - a^2)\int \sin 2x\,dx \quad \Rightarrow \frac{-1}{y} = (b^2 - a^2)\left[\frac{-\cos 2x}{2}\right]$

$\therefore y = \frac{2}{(b^2 - a^2)\cos 2x} = f(x)$

45. If $\int \frac{\sqrt{x}}{x(x+1)}dx = k\,\mathrm{Tan}^{-1} m$, then (k, m) is ________

(1) $(1, \sqrt{x})$ (2) $(2, \sqrt{x})$ (3) (2, x) (4) (1, x)

Ans (2)

$I = \int \frac{\sqrt{x}}{x(x+1)}dx$

$$= \int \frac{dx}{\sqrt{x}\ (x+1)} = 2\int \frac{\frac{1}{2\sqrt{x}}}{\left[\left(\sqrt{x}\right)^2+1\right]} dx$$

Let $\sqrt{x} = t \Rightarrow \frac{dx}{2\sqrt{x}} = dt$

$\therefore\ I = 2\int \frac{dt}{t^2+1} = 2\tan^{-1} t$

$\therefore\ I = 2\tan^{-1}\sqrt{x}\ = k\tan^{-1} m$

$\therefore\ (k,\ m) = \left(2,\ \sqrt{x}\right)$

46. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then $\frac{dy}{dx} =$ ________

(1) $-\sqrt[3]{\frac{x}{y}}$ (2) $-\sqrt[3]{\frac{y}{x}}$ (3) $\sqrt[3]{\frac{y}{x}}$ (4) $\sqrt[3]{\frac{x}{y}}$

Ans (2)

$x = a\cos^3\theta\ ;\ y = a\sin^3\theta \ \Rightarrow x^{2/3} + y^{2/3} = a^{2/3}$

$$\Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0$$

$$\Rightarrow y' = -\left(\frac{x}{y}\right)^{-\frac{1}{3}} \quad \Rightarrow y' = -\sqrt[3]{\frac{y}{x}}$$

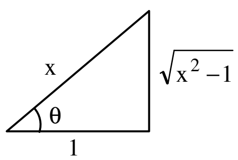
47. If $y = \tan^{-1}\sqrt{x^2-1}$, then the ratio $\frac{d^2y}{dx^2} : \frac{dy}{dx} =$ ________

(1) $\frac{1+2x^2}{x\ (x^2+1)}$ (2) $\frac{x\,(x^2+1)}{1-2x^2}$ (3) $\frac{x(x^2-1)}{1+2x^2}$ (4) $\frac{1-2x^2}{x\,(x^2-1)}$

Ans (4)

$y = \tan^{-1}\sqrt{x^2-1}$

Let $\tan\theta = \sqrt{x^2-1}$



$\therefore\ \sec\theta = x \quad$ or $\quad y = \sec^{-1} x$

$\therefore\ \frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$

(or) $\frac{dy}{dx} = (x^4 - x^2)^{-\frac{1}{2}}$

$$\therefore\ \frac{d^2y}{dx^2} = -\frac{1}{2}(x^4-x^2)^{-\frac{3}{2}}(4x^3-2x)$$

$$= -\frac{1}{2}\frac{2x(2x^2-1)}{x^3(x^2-1)\sqrt{x^2-1}}$$

$$= \frac{-(2x^2-1)}{x^2(x^2-1)\sqrt{x^2-1}}$$

$$\therefore \frac{d^2y}{dx^2} : \frac{dy}{dx} = \frac{-(2x^2-1)}{x(x^2-1)} \cdot \frac{1}{x\sqrt{x^2-1}} : \frac{1}{x\sqrt{x^2-1}}$$

$$= \frac{1-2x^2}{x(x^2-1)}$$

48. P is the point of contact of the tangent from the origin to the curve y = $\text{Log}_e x$. The length of the perpendicular drawn from the origin to the normal at P is ________

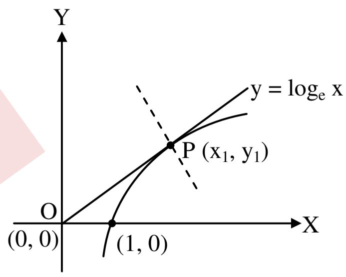
(1) $2\sqrt{e^2+1}$ (2) $\sqrt{e^2+1}$ (3) $\frac{1}{2e}$ (4) $\frac{1}{e}$

Ans (2)

$y = \log x$ … (1)

$\therefore \frac{dy}{dx} = \frac{1}{x_1}$

$\therefore (y - y_1) = \frac{1}{x_1}(x - x_1)$ … (2)



(2) passes through origin $\Rightarrow$ put y = x = 0

$\Rightarrow -y_1 = \frac{1}{x_1}(-x_1)$ $\therefore y_1 = 1 \Rightarrow x_1 = e$

$\therefore P(x_1, y_1) = (e, 1)$

$\therefore OP = \frac{y_1}{(y)^2}\sqrt{1+(y')^2}$

$= \frac{1}{1/e}\sqrt{1+\frac{1}{e^2}} = e\sqrt{\frac{e^2+1}{e}}$ $\therefore OP = \sqrt{e^2+1}$

Aliter:

Let $y = \log_2 x$ …(1)

$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$

Slope of T : $m = \frac{1}{x}$ $\therefore \left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} = \frac{1}{x_1} = m$

$\therefore OP = \sqrt{1+e^2}$

Let $P \equiv (x_1, y_1)$

y = mx is the equation of T

$\Rightarrow y_1 = \frac{1}{x_1}(x_1) \Rightarrow y_1 = 1$

$\therefore (1) \Rightarrow x_1 = e$

$P \equiv (e, 1)$

49. For the curve $4x^5 = 5y^4$, the ratio of the cube of the subtangent at the point on the curve to the square of the subnormal at the same point is ________

(1) $\left(\frac{4}{5}\right)^4$ (2) $\left(\frac{5}{4}\right)^4$ (3) $x\left(\frac{4}{5}\right)^4$ (4) $y\left(\frac{5}{4}\right)^4$

Ans (1)

Given $4x^5 = 5y^4$ … (1)

$\Rightarrow\ (4)5\cdot x^4 = (5)4\cdot y^3\dfrac{dy}{dx}$

$\therefore\ \dfrac{dy}{dx} = \dfrac{x_1^4}{y_1^3}\cdot \text{at}(x_1, y_1)$

(i) $(ST)^3 = \left[\dfrac{y_1}{x_1^4 / y_1^3}\right]^3 = \left[\left(\dfrac{y_1}{x_1}\right)^4\right]^3$

$\therefore\ (ST)^3 = \left(\dfrac{y_1}{x_1}\right)^{12}$

(ii) $(SN)^2 = \left[y_1 \times \dfrac{x_1^4}{y_1^3}\right]^2 = \dfrac{x_1^8}{y_1^4}$

$\therefore\ \dfrac{(ST)^3}{(SN)^2} = \dfrac{(y_1/x_1)^{12}}{x_1^8/y_1^4}$

$= \dfrac{y_1^{16}}{x_1^{20}} = \left[\dfrac{y_1^4}{x_1^5}\right]^4$

$\therefore\ \dfrac{(ST)^3}{(SN)^2} = \left(\dfrac{4}{5}\right)^4$

50. The set of real values of x for which $f(x) = \dfrac{x}{\text{Log}x}$ is increasing, is ________

(1) $\{x : x < e\}$ (2) $\{1\}$ (3) $\{x : x \geq e\}$ (4) empty

Ans (3)

$f(x) = \dfrac{x}{\log x}$

$\therefore\ f'(x) = \dfrac{1}{(\log x)^2}\left[\log x(1) - \dfrac{x}{x}\right]$

$= \dfrac{\log x - 1}{(\log x)^2}$

$\therefore\ f'(x) > 0$

$\Rightarrow \log x - 1 > 0\quad \therefore \log x > 1\quad \text{or } x > e$

$\Rightarrow\ \{x : x \geq e\}$

51. $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x}\,dx =$ ________

(1) $\dfrac{1}{2}\log 3$ (2) $2\log 3$ (3) $\dfrac{1}{4}\log 3$ (4) $\log 3$

Ans (3)

Given $I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} \cdot dx$

$(\sin x - \cos x)^2 = 1 - \sin 2x$

(or) $\sin 2x = 1 - (\sin x - \cos x)^2$

$\therefore \; I = \int_0^{\frac{\pi}{4}} \frac{d\,(\sin x - \cos x)}{4 - (\sin x - \cos x)^2} = \int_0^{\frac{\pi}{4}} \frac{d(\sin x - \cos x)}{2^2 - (\sin x - \cos x)^2}$

$= \frac{1}{4}\log_e \left[\frac{2 + (\sin x - \cos x)}{2 - (\sin x - \cos x)}\right]\Bigg|_0^{\frac{\pi}{4}}$

$= \frac{1}{4}\left[\log_e \left[\frac{2 + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)}{2 - \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)}\right] - \log_e \left[\frac{2 + (0-1)}{2 - (0-1)}\right]\right]$

$= \frac{1}{4}\left[0 - \log_e \frac{1}{3}\right] = \frac{1}{4}\,(-1)\,(-1)\,\log_e (3)$

$\therefore \; I = \frac{1}{4}\log_e 3$

52. $\int_0^1 x\,(1-x)^{\frac{3}{2}}\,dx =$ ________

(1) $\frac{24}{35}$ (2) $\frac{-8}{35}$ (3) $\frac{-2}{35}$ (4) $\frac{4}{35}$

Ans (4)

$I = \int_0^1 x\,(1-x)^{\frac{3}{2}} \cdot dx$

$= \int_0^1 (1-x)\,x^{\frac{3}{2}}\,dx = \int_0^1 x^{\frac{3}{2}} \cdot dx - \int_0^1 x^{\frac{5}{2}} \cdot dx$

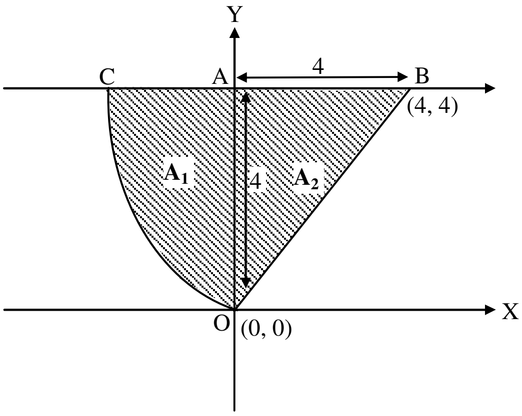
$\therefore \; I = \left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1 = \frac{2}{5} - \frac{2}{7} = \frac{4}{35}$ $\qquad \therefore \; I = \frac{4}{35}$

53. The area bounded by the curve $y = \begin{cases} x^2, & x < 0 \\ x, & x \ge 0 \end{cases}$ and the line $y = 4$ is ________

(1) $\frac{40}{3}$ (2) $\frac{16}{3}$ (3) $\frac{32}{3}$ (4) $\frac{8}{3}$

Ans (1)

Given $y = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$: $y = 4$



$\therefore$ $A = A_1 + A_2$

A_2 = area of triangle OAB $= \frac{1}{2}(4)(4)$

$A_2 = 8$ sq. units.

$$A_1 = \int_0^4 \sqrt{y} \cdot dy = \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{2^3}{\frac{3}{2}} = \frac{2^4}{3}$$

$\therefore$ $A = \frac{16}{3} + 8$

$A = \frac{40}{3}$ sq. units. $\therefore$ $A_1 = \frac{16}{3}$ sq. units.

54. The order and degree of the differential equation $y = \frac{dp}{dx}x + \sqrt{a^2p^2 + b^2}$ where $p = \frac{dy}{dx}$ (here a and b are arbitrary constants) respectively are ________

(1) 1, 2 (2) 2, 1 (3) 2, 2 (4) 1, 1

Ans (3)

Given $y = \frac{dp}{dx}x + \sqrt{a^2p^2 + b^2}$

$\forall p = \frac{dy}{dx} \Rightarrow \frac{dp}{dx} = \frac{d^2y}{dx^2}$

$\therefore \left[y - \frac{d^2y}{dx^2}x\right]^2 = a^2\left(\frac{dy}{dx}\right)^2 + b^2$

$\therefore$ order = 2, degree = 2.

55. The general solution of the differential equation $2x\frac{dy}{dx} - y = 3$ is a family of ________

(1) straight lines (2) circles (3) hyperbolas (4) parabolas

Ans (4)

Given $2x \cdot \frac{dy}{dx} - y = 3$

$\therefore$ $2x \cdot \frac{dy}{dx} = (3 + y)$

$\int \frac{dy}{3+y} = \int 2x \cdot dx \Rightarrow \log_e(3+y) = \frac{1}{2}\log_e 2x + \log c$

$\therefore$ $(3+y) = \sqrt{2x} \cdot c$

$(3+y)^2 = (2x)c^1 \Rightarrow$ family of parabolas.

56. A wire of length 20 cm is bent in the form of a sector of a circle. The maximum area that can be enclosed by the wire is ________

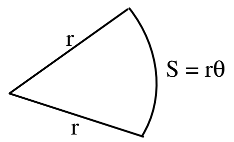
(1) 10 sq. cm (2) 30 sq. cm (3) 20 sq. cm (4) 25 sq. cm

Ans (4)

Given : Perimeter r = 20 cm

$\Rightarrow P = r + r + r\theta = r(2 + \theta) = 20$

$\therefore r(2 + \theta) = 20$ ...(1)



We know that for area to be max $\Rightarrow r = \frac{P}{4} \Rightarrow r = \frac{20}{4} = 5$

(1) $\Rightarrow r(2 + \theta) = 20$

$\therefore 2 + \theta = 4$ or $\theta = 2$

$\therefore r = 5$ cm ; $\theta = 2^1$ $\therefore A = \frac{1}{2} r^2\theta = \frac{1}{2}(2)(5)^2$ $\therefore A = 25$ sq. cm.

57. Two circles centered at (2, 3) and (5, 6) intersect each other. If the radii are equal, the equation of the common chord is ________

(1) $x + y - 8 = 0$ (2) $x - y - 8 = 0$ (3) $x + y + 1 = 0$ (4) $x - y + 1 = 0$

Ans (1)

Let the equations be

S_1: $(x-2)^2 + (y-3)^2 = r^2$, S_2 : $(x - 5)^2 + (y - 6)^2 = r^2$ as radii are equal

$\therefore$ R.A = ?

$4 - 4x + 9 - 6y - 25 + 10x - 36 + 12y = 0$

$6x + 6y - 48 = 0$ or $x + y - 8 = 0$

58. Equation of the circle centered at (4, 3) touching the circle $x^2 + y^2 = 1$ externally, is ________

(1) $x^2 + y^2 + 8x - 6y + 9 = 0$. (2) $x^2 + y^2 - 8x + 6y + 9 = 0$

(3) $x^2 + y^2 - 8x - 6y + 9 = 0$. (4) $x^2 + y^2 + 8x + 6y + 9 = 0$

Ans (3)

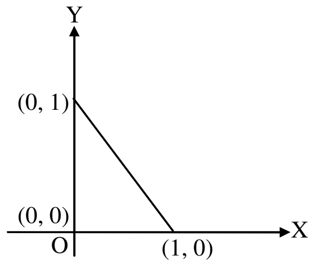
By inspection ; centre $\equiv$ (4, 3) satisfies the equation in (3).

59. The points (1, 0), (0, 1), (0, 0) and (2k, 3k), $k \neq 0$ are concyclic if k = ________

(1) $-\frac{5}{13}$ (2) $\frac{5}{13}$ (3) $\frac{1}{5}$ (4) $-\frac{1}{5}$

Ans (2)

Equation of circle $x(x - 1) + y(y - 1) = 0$ ...(1)



(2k, 3) lies on 91) ;

$2k(2k - 1) + 3k(3k - 1) = 0$

as $k \neq 0 \Rightarrow 2(2k - 1) + 3(3k - 1) = 0$

$4k + 9k - 2 - 3 = 0$

$\therefore k = \frac{5}{13}$

60. The locus of the point of intersection of the tangents drawn at the ends of a focal chord of the parabola $x^2 = -8y$ is ________

(1) $y = 2$ (2) $y = -2$ (3) $x = 2$ (4) $x = -2$

Ans (1)

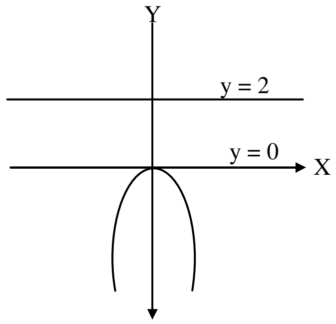
Equation of directrix = ?

$x^2 = -8y$; $x^2 = -4ay$

$\Rightarrow a = 2.$

Equation of directrix $y = a$

$\therefore y = 2.$



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