

MATHEMATICS

Class X Standard – CBSE
Solution to Set - 1

SECTION - A

1. If $\frac{p}{q}$ is a rational number in its standard form, then the decimal representation of $\frac{p}{q}$ is a terminating decimal only when q is of the form $2^m \times 5^n$, for some non negative integers m and n .

2. The n th term $t_n = 6n + 2$

$$\Rightarrow t_1 = 6 + 2 = 8, t_2 = 6 \times 2 + 2 = 14$$

$$\therefore d = t_2 - t_1 = 14 - 8 = 6$$

3. $\sec A = \frac{1}{\cos A} \therefore (\sin A + \cos A) \frac{1}{\cos A}$
 $= \frac{\sin A}{\cos A} + \frac{\cos A}{\cos A}$
 $= \tan A + 1 = \frac{5}{12} + 1 = \frac{17}{12}$.

4. 3

5. $\angle QOR = 153^\circ$, $\angle ORP$ is $\angle OQP = 90^\circ$ (Radius and the tangent at the point of contact are \perp to each other).

\therefore In Quadrilateral $OQPR$.

$$\angle ORP + \angle P + \angle OQP + \angle O = 360^\circ.$$

$$90^\circ + 153^\circ + 90^\circ + \angle P = 360^\circ.$$

$$\angle P + 333^\circ = 360^\circ$$

$$\angle P = 360 - 333$$

$$\angle P = 27^\circ.$$

6. $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sin \theta = \tan \theta \times \cos \theta$

$$= \tan \theta \times \sqrt{\frac{1}{\sec^2 \theta}}$$

$$= \tan \theta \times \sqrt{\frac{1}{1 + \tan^2 \theta}} = \tan \theta \times \frac{1}{\sqrt{1 + \tan^2 \theta}} = \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}$$

7. To have equal roots

$$D = b^2 - 4ac = 0 \quad a = k, b = -5, c = k$$

$$(-5)^2 - (4 \times k \times k) = 0$$

$$25 - 4k^2 = 0$$

$$25 = 4k^2$$

$$k^2 = \frac{25}{4} \text{ or } k = \pm \sqrt{\frac{25}{4}}$$

$$k = \pm \frac{5}{2}$$

$$\begin{aligned} 8. \text{ length of the arc} &= \frac{\theta}{360} \times 2\pi r \\ &= \frac{30}{360} \times 2 \times \frac{22}{7} \times 21 \\ &= 11 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{perimeter of the sector} &= 21 + 21 + 11 \\ &= 53 \text{ cm.} \end{aligned}$$

$$9. T_n = a + (n - 1)d \quad a = 2, \quad d = 5 \quad T_{10} = ?$$

$$t_{10} = 2 + (10 - 1)5$$

$$t_{10} = 2 + 9 \times 5 = 2 + 45 = 47$$

$$10. \text{ Total no. of events} = 35$$

No. of events favourable to E = 7.

i.e., {5, 10, 15, 20, 25, 30, 35}

$$P(E) = \frac{\text{No. of events favourable to E}}{\text{Total no. of events}}$$

$$= \frac{7}{35} = \frac{1}{5}$$

$$11. x^2 + px - 4 = 0$$

of the equation = -4

$$(-4)^2 + (px - 4) - 4 = 0$$

$$16 - 4p - 4 = 0$$

$$-4p = 4 - 16$$

$$4p = 12$$

$$p = \frac{12}{4}$$

$$p = 3$$

Or

$$f(x) = x^2 - 1$$

$$x^2 + 0x - 1 = 0$$

$$\text{sum of the zeros } [\alpha + \beta] = \frac{-b}{a}$$

$$= \frac{-0}{1}$$

$$= 0$$

SECTION - B

Substituting the value of p in the equation $x^2 + px + k$

$$x^2 + px + k = 0, \quad p = 3$$

$$x^2 + 3x + k = 0$$

If it has real roots:

$$b^2 - 4ac = 0$$

$$(3)^2 - (4 \times 1 \times k) = 0$$

$$9 - 4k = 0$$

$$+4k = 9$$

$$k = \frac{9}{4}$$

sum of the zeroes:

$$\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$$

$$= \frac{2(\alpha^2 + \beta^2)}{\alpha\beta} = \frac{2[(\alpha + \beta)^2 - 2\alpha\beta]}{\alpha\beta}$$

$$= \frac{2[(0)^2 - (2 \times (-1))]}{-1}$$

$$= \frac{4}{-1} = -4$$

product of the zeros $[\alpha\beta] = \frac{c}{a} = -1$

$$\left(\frac{2\alpha}{\beta}\right)\left(\frac{2\beta}{\alpha}\right) = 4$$

Required polynomial = $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 + 4x + 4$

12. We have

$$\sin(A + B) = 1 \qquad \cos(A - B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin(A + B) = \sin 90^\circ \text{ and } \Rightarrow \cos(A - B) = \cos 30^\circ$$

$$\Rightarrow A + B = 90^\circ \dots(1) \qquad \Rightarrow A - B, 30^\circ \dots(2)$$

Adding (1) and (2)

$$\begin{array}{ll} A + B = 90^\circ & A + B = 90^\circ \\ A - B = 30^\circ & B = 90 - A \\ 2A = 120 & B = 90 - 60 = 30^\circ \\ A = \frac{120}{2} = 60^\circ & \therefore \angle A = 60^\circ, \angle B = 30^\circ. \end{array}$$

13. The capacity of the container is 60l. [Solution to be given].

14. $a_1 = 3 \qquad b_1 = 2 \qquad c_1 = -5$

$a_2 = 2 \qquad b_2 = -3 \qquad c_2 = -7$

equation = $3p + 2q = 5, 2p - 3q = 7$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{3}{2} \neq \frac{-2}{3}$$

\therefore The given equation is intersecting and hence it is consistent.

15. The end points of AB are

A(4, -3) and B(9, 7)

$\therefore (x_1 = 4, y_1 = -3)$ are A(4, -3) and B(9, 7)

Also $m = 3$ and $n = 2$

let the required point be p(x, y)

By section formula, we have

$$\frac{x}{m+n} = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

$$\Rightarrow \frac{x}{3+2} = \frac{(3 \times 9) + (2 \times 4)}{3+2}, \quad y = \frac{(3 \times 7) + (2 \times -3)}{3+2}$$

$$\Rightarrow x = 7, y = 3$$

Hence the required point is p(7, 3)

16. The given system of equations may be written as $x + y - (a - b) = 0$.

$ax - by - (a^2 + b^2) = 0$. By cross multiplication

we have, $\frac{x}{-(a^2 + b^2) - b(a - b)} = \frac{y}{-a(a - b) + (a^2 + b^2)} = \frac{1}{-b - a}$

$$\Rightarrow \frac{x}{-a^2 - ab} = \frac{y}{ab + b^2} = \frac{1}{-b - a}$$

$$\Rightarrow \frac{x}{-a(a + b)} = \frac{y}{b(a + b)} = \frac{1}{-(a + b)}$$

$$\Rightarrow x = \frac{-a(a + b)}{-(a + b)} = a \text{ and } y = \frac{b(a + b)}{-(a + b)} = -b$$

Hence $x = a, y = -b$ is the solution of the given system of equations.

17. The given equations are $3x + y - 1$

$$(2k - 1)x + (k - 1)y - (2k + 1) = 0$$

These equations are of the form

$$a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0.$$

where $a_1 = 3, b_1 = 1, c_1 = -1$

$$a_2 = (2k - 1), b_2 = (k - 1), c_2 = -(2k + 1)$$

For no solution, we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\text{Now } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3}{2k - 1} = \frac{1}{k - 1} \neq \frac{-1}{-(2k + 1)} \Rightarrow \frac{3}{(2k - 1)} = \frac{1}{k - 1} \text{ \& } \frac{1}{k - 1} \neq \frac{1}{2k + 1}$$

$$\Rightarrow 3k - 3 = 2k - 1 \text{ \& } (2k + 1) \neq k - 1$$

$$\Rightarrow k = 2, \text{ \& } k \neq -2.$$

Thus $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ holds when $k = 2$.

Hence the given system of equations has no solution when $k = 2$.

OR

$$\frac{57}{x + y} + \frac{6}{x - y} = 5, \quad \frac{38}{x + y} + \frac{21}{x - y} = 9$$

= let $\frac{1}{x + y} = p$ \& $\frac{1}{x - y} = q$, the given equations becomes

$$57p + 6q = 5$$

$$33p + 21q = 9$$

Multiplying (i) by 2 \& (ii) by 3 we get

$$114p + 12q = 10$$

$$q = \frac{1}{3}$$

On subtracting $114p + 63q = 27$

$$114p + \left(12 \times \frac{1}{2}\right) = 10$$

$$-51q = -17$$

$$114p = 6$$

$$\therefore q = \frac{p}{3}$$

$$p = \frac{1}{19}$$

$$\therefore p = \frac{1}{19} = \frac{1}{x+y}$$

$$q = \frac{1}{3} = \frac{1}{x-y}$$

$$\therefore x + y = 19 \quad \dots(1)$$

$$\therefore x - y = 3 \quad \dots(2)$$

Adding (1) and (2)

$$2x = 22$$

$$x = 11$$

$$\therefore x + y = 19$$

$$\therefore 11 + y = 19$$

$$y = 19 - 11$$

$$y = 8$$

$$\therefore x = 11, y = 8$$

18. Let a be given +ve odd integer. In dividing a by 6, let q be the quotient and r be the remainder.

Then by Euclid's division lemma, we have

$$a = bq + r \text{ where } 0 \leq r < 6.$$

$$\Rightarrow a = 6q + r \text{ where } r = 0, 1, 2, 3, 4, 5$$

$$\Rightarrow \text{case (i) } r = 0$$

$$a = 6q + r = 6q + 0$$

$$\therefore a = 6q = 2(3q) \text{ is an even integer.}$$

$$r = 1, \quad a = 6q + 1 \quad \text{not an even integer.}$$

$$r = 2 \quad a = 6q + 2 = 2(3q + 1) \text{ is an even integer.}$$

$$r = 3 \quad a = 6q + 3 = 3 \text{'s not an even integer.}$$

$$r = 4 \quad a = 6q + 4 = 2(3q + 2) \text{ is an even integer.}$$

$$r = 5 \quad a = 6q + 5 \quad \text{is not an even integer.}$$

$\therefore 6q + 1, 6q + 3, 6q + 5$ are odd integers for some integer of when a is odd.

19. Let D be the mid-point of BC so that AD is a median of ΔABC coordinates of D are

$$\left[\frac{3+5}{2}, \frac{-2+2}{2} \right] \text{ i.e., } [4, 0]$$

$$\text{For } \Delta ABD, x_1 = 4, y_1 = -6, x_2 = 3, y_2 = -2, x_3 = 4, y_3 = 0$$

\therefore area of ΔABD .

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [4(-2 - 0) + 3(0 + 6) + 4(-6 + 2)]$$

$$= \frac{1}{2} [-8 + 18 - 16] = \frac{1}{2} \times (-6) = -3 \text{ sq. units}$$

$$= 3 \text{ sq. units}$$

$$\text{For } \Delta ACD, x_1 = 4, y_1 = -6, x_2 = 5, y_2 = 2, x_3 = 4, y_3 = 0$$

$$\text{Area of } \Delta ACD = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

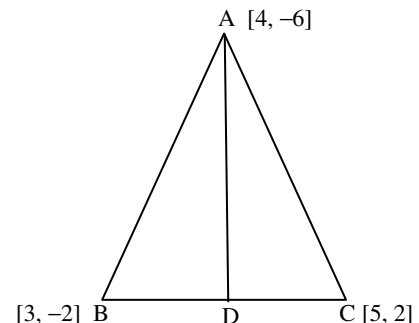
$$= \frac{1}{2} [4(2 - 0) + 5(0 + 6) + 4(-6 - 2)]$$

$$= \frac{1}{2} [8 + 30 - 32] = \frac{1}{2} \times 6 = 3 \text{ sq. units, which verifies the result.}$$

Or

ABCD is a parallelogram.

Let the diagonals intersect each other at O.



We know that the diagonals of a parallelogram bisect each other coordinates of the mid-point of AC are

$$\left[\frac{-2+a}{2}, \frac{-1+3}{2} \right] \text{ i.e., } \left[\frac{-2+a}{2}, 1 \right]$$

Coordinates of the mid-point of BD are $\left[\frac{1+1}{2}, \frac{0+b}{2} \right]$ i.e., $\left[1, \frac{b}{2} \right]$

But $\left[\frac{-2+a}{2}, 1 \right] = \left[1, \frac{b}{2} \right]$

$$\Rightarrow \frac{-2+a}{2} = 1 \text{ and } 1 = \frac{b}{2}$$

$$\Rightarrow a = 2 + 2 = 4, b = 2$$

$$\therefore a = 4, b = 2$$

20. Construct to be shown.

21. We have

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{x-(a+b+x)}{x(a+b+x)} = \frac{b+a}{ab}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{-ab}{x(a+b+x)} = \frac{a+b}{a-b} = 1$$

$$\Rightarrow \frac{-1}{x(a+b+x)} = \frac{1}{ab} \text{ [on dividing both sides by (ab)]}$$

$$x(a+b+x) = -ab$$

$$ax + bx + x^2 + ab = 0$$

$$x^2 + ax + bx + ab = 0$$

$$x(x+a)b(x+a) = 0$$

$$(x+a)(x+b) = 0$$

$$x+a=0, x+b=0$$

$$\Rightarrow x = -a, x = -b$$

Hence $-a$ and $-b$ are the roots of the given equation.

OR

The given equation is

$$a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$$

Comparing it with $ax^2 + bx + c = 0$, we get

$$a = a^2b^2, b = -(4b^4 - 3a^4), c = -12a^2b^2$$

$$\Rightarrow b = 3a^4 - 4b^4$$

$$\therefore D = (b^2 - 4ac)$$

$$= (3a^4 - 4b^4)^2 - 4a^2b^2(-12a^2b^2)$$

$$= 9a^8 + 16b^8 - 24a^4b^4 + 48a^4b^4 = 9a^8 + 16b^8 + 24a^4b^4$$

$$= (3a^4)^2 + (4b^4)^2 + 2 \times 3a^4 \times 4b^4 = (3a^4 + 4b^4)^2 \geq 0$$

Hence the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{(4b^4 - 3a^4) + (3a^4 + 4b^4)}{2a^2b^2} = \frac{8b^4}{2a^2b^2} = \frac{4b^2}{a^2}$$

$$\text{and } \beta = \frac{-b - \sqrt{D}}{2a} = \frac{(4b^4 - 3a^4) - (3a^4 + 4b^4)}{2a^2b^2} = \frac{-6a^4}{2a^2b^2} = \frac{-3a^2}{b^2}$$

hence, $\frac{4b^2}{a^2}$ and $\frac{-3a^2}{b^2}$ are the roots of the given equation.

$$\begin{aligned} 22. \quad & \frac{\tan^2 60^\circ + 4\sin^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ} \\ &= \frac{(\sqrt{3})^2 + 4 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \left(\frac{2}{\sqrt{3}}\right)^2 + 5 \times 0^2}{2 + 2 - (\sqrt{3})^2} = \frac{3 + 4 \times \frac{1}{2} + 3 \times \frac{4}{3} + 5 \times 0}{4 - 3} \\ &= \frac{3 + 2 + 4 + 0}{4 - 3} = \frac{9}{1} = 9 \end{aligned}$$

OR

$$\begin{aligned} & \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta \\ \text{LHS: } & \left[\frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} \right] + \left[\frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \right] \\ &= \frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{(\sin \theta - \cos \theta)} + \frac{\cos \theta}{\sin \theta} \times \frac{\cos \theta}{(\cos \theta - \sin \theta)} \\ &= \frac{\sin^2 \theta}{\cos \theta(\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta(\sin \theta - \cos \theta)} \\ &= \frac{(\sin^3 \theta - \cos^3 \theta)}{\sin \theta \cos \theta(\sin \theta - \cos \theta)} \\ &= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta(\sin \theta - \cos \theta)} \\ &= \frac{\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta}{\sin \theta \cos \theta} \\ &= \frac{1 + \sin \theta \cos \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} + 1 = 1 + \frac{1}{\cos \theta \sin \theta} \\ &= (1 + \sec \theta \operatorname{cosec} \theta) = \text{RHS} \end{aligned}$$

$$23. \quad BC^2 = AC \times CD$$

$$\Rightarrow BC \times BC = AC \times CD$$

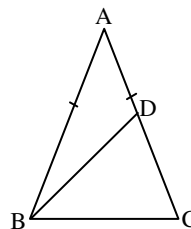
$$\frac{BC}{AC} = \frac{DC}{BC} \quad \dots (i)$$

In $\triangle ABC$ and $\triangle BDC$

$$\Rightarrow \frac{BC}{CA} = \frac{DC}{CB} \text{ and } \angle C = \angle C \text{ [common]}$$

By SAS similarity

$$\triangle ABC \sim \triangle BDC$$



$$\frac{AB}{BD} = \frac{BC}{DC}$$

$$\frac{AC}{BD} = \frac{BC}{DC} \quad [\because AB = AC]$$

$$\frac{AC}{BC} = \frac{BD}{CD} \quad \dots \text{(ii)}$$

$$\frac{BC}{CD} = \frac{BD}{CD}$$

$$\therefore BD = BC$$

24. Let a be the first term and 'd' be the common difference of the AP. We have,

$$t_3 + t_7 = 6 \text{ and } t_3 t_7 = 8$$

$$\Rightarrow (a + 2d) + (a + 6d) = 6 \text{ and } (a + 2d)(a + 6d) = 8$$

$$2a + 8d = 6 \text{ and } (a + 2d)(a + 6d) = 8$$

$$a + 4d = 3 \text{ and } (a + 2d)(a + 6d) = 8$$

$$a = 3 - 4d \text{ and } (a + 2d)(a + 6d) = 8$$

$$(3 - 4d + 2d)(3 - 4d + 6d) = 8. \quad [\text{putting } a = 3 - 4d \text{ in the second equation}]$$

$$(3 - 2d)(3 + 4d) = 8$$

$$\Rightarrow 9 - 4d^2 = 8$$

$$\therefore 4d^2 = 1 \quad \text{or} \quad d^2 = \frac{1}{4} \quad \text{or} \quad d = \pm \frac{1}{2}$$

where $d = \frac{1}{2}$, $a = 3 - 4d$, we get

$$a = 3 - 4 \times \frac{1}{2} = 3 - 2 = 1$$

$$\therefore S_{16} = \frac{n}{2} [2a + (n-1)d]$$

$$S_{16} = \frac{16}{2} \left[2 \times 1 + \left(15 \times \frac{1}{2} \right) \right]$$
$$= 8 \times \frac{19}{2} = 76$$

When $d = -\frac{1}{2}$, $a = 3 - 4d$ we get $a = 3 + 2 = 5$

$$\therefore S_{16} = \frac{16}{2} \left[2a + (16-1) \frac{1}{2} \right]$$

$$S_{16} = 8 \left[10 + 15 \times -\frac{1}{2} \right]$$

$$S_{16} = 8 \times \frac{5}{2} = 20$$

25. Solution to be given

26.

CI	Class mark (x_i)	Frequency (f_i)	Deviations $u_i = \frac{x_i - a}{h}$	$f_i u_i$
0 – 10	5	10	-3	-30
10 – 20	15	15	-2	-30
20 – 30	25	60	-1	-60
30 – 40	35	55	0	0
40 – 50	45	50	1	50
50 – 60	55	10	2	20

Let the assumed mean $a = 35$, $h = 10$, $\Sigma f_i u_i = -50$

Mean $\bar{x} = a + h\bar{u}$

$$= a + h \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 35 + \left[\frac{-50}{200} \times 10 \right]$$

$$= 35 - 2.5 = 32.5, \text{ Hence avg. age} = 32.5 \text{ years.}$$

OR

Heights (in cm)	No. of students (f_i)	C_f
135 – 140	4	4
140 – 145	9	13
145 – 150	18	31
150 – 155	11	42
155 – 160	6	48
160 – 165	5	53

Here $n = 53$

$$\frac{n}{2} = \frac{53}{2} = 26.5$$

Now 26.5 belongs to 145 – 150 class

\therefore The median lies in the class 145 – 150.

$\therefore l = 145$, $f = 18$, $C = 13$ and $h = 5$

$$\therefore \text{Median} = L + \frac{\left(\frac{n}{2} - c \right)}{f} \times h$$

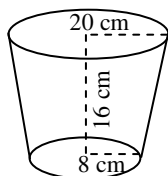
$$= 145 + \frac{(26.5 - 13)}{18} \times 5$$

$$= 145 + \frac{13.5 \times 5}{18} = 145 + \frac{67.5}{18} = 145 + 3.75 = 148.75$$

Hence median = 148.75

27. Volume of milk = Volume of bucket

$$= \frac{\pi h}{3} (R^2 + Rr + r^2) \text{ where } h = 16 \text{ cm, } R = 20 \text{ cm, } r = 8 \text{ cm, } V = \frac{\pi \times 16}{3} (20^2 + 20 \times 8 + 8^2) \text{ cm}^3$$



$$\frac{3.14 \times 16}{3} (400 + 160 + 64) \text{cm}^3 = \frac{3.14 \times 16 \times 624}{3} \text{cm}^3 = 10449.92 \text{cm}^3$$

$$= 10.44992 \text{ litres } \dots (1000 \text{ cm}^3 = 1 \text{ l}).$$

∴ cost of 1 litre of milk = Rs. 20.

Cost of 10.44992 litres of milk = Rs. 10.44992 × 20

= Rs. 208.9984 = Rs. 209.

Whole surface area = curved surface area + surface area of bottom = $\pi l(R + r) + \pi r^2$.

Here $l = \sqrt{h^2 + (R - r)^2} = \sqrt{(16)^2 + (12)^2} \text{cm}$.

= $\sqrt{256 + 144} \text{cm} = \sqrt{400} \text{cm} = 20 \text{cm}$.

Whole surface area = $[\pi \times 20(20 + 8) + \pi(8)^2] \text{cm}$.

= $\pi[560 + 64] \text{cm}^2 = 624 \pi \text{cm}^2 = 6.24 \pi \text{dm}^2 = 6.24 \times 3.14 \text{dm}^2$.

= 19.5936 dm^2 .

∴ cost of 1 dm^2 of metal = Rs. 8.

Cost of 19.5936 dm^2 of metal = Rs. 19.5936 × 8 = Rs. 156.7488

= Rs. 156.75

OR

Let ABC be the right angled triangle in which $\angle A = 90^\circ$. Hypotenuse

$BC = \sqrt{AB^2 + AC^2} = \sqrt{15^2 + 20^2} \text{cm}$.

= $\sqrt{225 + 400} \text{cm} = \sqrt{625} \text{cm} = 25 \text{cm}$.

Let AO or A'O be the radius of the common base of the double cone formed by revolving the triangle about BC. Height of the cone CAA' is CO and slant height AC = 20 cm. Height of the cone BAA' is BO and slant height AB = 15 cm.

Now $\Delta AOB \sim \Delta CAB$

$\frac{AO}{CA} = \frac{AB}{CB}$ or $\frac{AO}{20} = \frac{15}{25}$ ∴ $AO = \frac{15 \times 20}{25} \text{cm} = 12 \text{cm}$.

From right angled ΔABO

$BO^2 + OA^2 = AB^2 \Rightarrow BO^2 = AB^2 - OA^2 = 15^2 - 12^2$

= $225 - 144.81 \Rightarrow BO = 9 \text{cm}$. $CO = BC - BO = (25 - 9) \text{cm} = 16 \text{cm}$.

Volume of double cone = volume (cone ABA' + cone CAA')

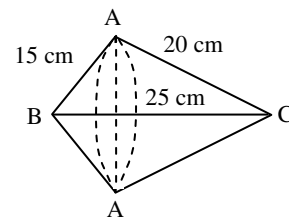
= $\left[\frac{1}{3} \pi (12)^2 \times 9 + \frac{1}{3} \pi (12)^2 \times 16 \text{cm}^3 \right]$

= $\frac{1}{3} \pi (12)^2 (9 + 16) \text{cm}^3 = \frac{1}{3} \times 3.14 \times 12^2 \times 25 \text{cm}^3 = 3768 \text{cm}^3$.

Surface area of double cone = surface area of (cone ABA' + cone CAA')

= $(\pi \times 2 \times 15 + \pi \times 12 \times 20) \text{cm}^2 = \pi \times 12 \times 35 \text{cm}^2$.

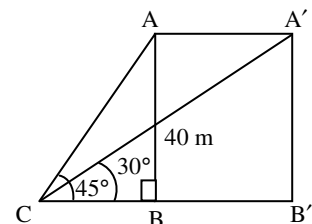
= $3.14 \times 12 \times 35 \text{cm}^2 = 1318.8 \text{cm}^2$.



28. Let AB = 40 m be height of the house.

A represents the crow. The angle of elevations at C is 45° .

Now from right. ΔABC , $\frac{AB}{BC} = \tan 45^\circ$, $\frac{AB}{BC} = 1$, $BC = AB = 40 \text{m}$.



Let the crow after 3 secs be at position A' at a height of 40m from ground i.e., A'B' = 40 m.

Suppose AA' = x metres. Elevation of A' at C is 30°,

From right $\triangle \Delta A'B'C$ $\frac{A'B'}{CB'} = \tan 30^\circ$.

$$\text{Or } \frac{40}{CB + BB'} = \frac{1}{\sqrt{3}} \quad \text{or } \frac{40}{40 + x} = \frac{1}{\sqrt{3}} \quad \text{or } 40\sqrt{3} = 40 + x.$$

$$\therefore x = 40(\sqrt{3} - 1) \quad \text{or } 40 \times 0.732 \text{ m} = 29.28 \text{ m}.$$

$$\text{Speed of the crow} = \frac{29.28}{3} \times \frac{60 \times 60}{1000} \text{ Km/hr} = 35.136 \text{ Km/hr}.$$

OR

Let AB be the tower and C and D be two objects such that CD = p metres. Let h metres be the height of the tower.

From right $\triangle \Delta ABD$,

$$\frac{h}{DB} = \tan \alpha \quad DB = \frac{h}{\tan \alpha} \quad \dots (i)$$

From right $\triangle \Delta ACB$ $\frac{h}{P + DB} = \tan \beta$ or $h = p \tan \beta + DB \tan \beta$

$$DB = \frac{h - p \tan \beta}{\tan \beta} \quad \dots (ii)$$

From (i) and (ii) we get

$$\frac{h}{\tan \alpha} = \frac{h - p \tan \beta}{\tan \beta} \Rightarrow h \tan \beta = h \tan \alpha - p \tan \alpha \tan \beta$$

$$h(\tan \alpha - \tan \beta) = p \tan \alpha \tan \beta$$

$$\text{Hence } \therefore h = \frac{p \tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$$

When $p = 50^\circ$, $\alpha = 60^\circ$ and $\beta = 30^\circ$.

$$h = \frac{50 \tan 60^\circ \tan 30^\circ}{\tan 60^\circ - \tan 30^\circ} = \frac{50 \times \sqrt{3} \times \frac{1}{\sqrt{3}}}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$= \frac{50\sqrt{3}}{3-1} = \frac{50\sqrt{3}}{2} = 25\sqrt{3}$$

\therefore Height of the tower = $25\sqrt{3}$ metres.

29. Suppose 1 man finishes the work in x days and 1 woman finishes the work in y days.

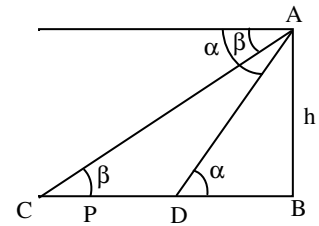
$$\therefore \text{work done by 2 women and 5 men in 1 day} = \frac{2}{y} + \frac{5}{x}.$$

Given 2 women and 5 men finish the work in 4 days. \therefore one day's work of 2 women and 5 men = $\frac{1}{4}$.

$$\therefore \frac{2}{y} + \frac{5}{x} = \frac{1}{4}, \quad \text{Again work done by 3 women and 6 men in 1 day} = \frac{3}{y} + \frac{6}{x}.$$

Given 3 women and 6 men finish the work in 3 days. \therefore one day's work of 3 women and 6 men = $\frac{1}{3}$

$$\therefore \frac{3}{y} + \frac{6}{x} = \frac{1}{3} \quad \dots (i)$$



$$\Rightarrow \frac{1}{y} + \frac{2}{x} = \frac{1}{9} \quad \dots \text{(ii)}$$

Multiplying (ii) by 2 we get

$$\frac{2}{y} + \frac{4}{x} = \frac{2}{9} \quad \text{sub (iii) from (i) we get}$$

$$\frac{1}{x} = \frac{1}{4} - \frac{2}{9} = \frac{9-8}{36} = \frac{1}{36} \quad x = 36$$

Putting $x = 36$ in (ii) we get

$$\frac{1}{y} + \frac{2}{36} = \frac{1}{9} \Rightarrow \frac{1}{y} + \frac{1}{18} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{9} - \frac{1}{18} = \frac{1}{18}$$

Hence 1000 man alone takes 18 days and 1 man alone takes 36 days to finish the work.

30. Given : $\triangle ABC$ in which $BC^2 = AB^2 + AC^2$, we need to prove that $\angle A = 90^\circ$.

Let us construct $\triangle DEF$ in which $DE = AB$, $DF = AC$, $\angle D = 90^\circ$.

Now in $\triangle DEF$, $\angle D = 90^\circ$

$$EF^2 = DE^2 + DF^2 = AB^2 + AC^2$$

(By Pythagoras theorem) (By construction)

But $BC^2 = AB^2 + AC^2$

$$\therefore EF^2 = BC^2 \text{ or } EF = BC$$

Now in $\triangle ABC$ and $\triangle DEF$

$AB = DE$, $AC = DF$, $BC = EF$ (prove)

$\therefore \triangle ABC = \triangle DEF$ (SSS congruency)

Hence $\angle A = \angle D = 90^\circ$ (CpeT)

Part II in $\triangle BCD$ $\angle C = 90^\circ$,

$$BD^2 = BC^2 + DC^2 \text{ (Pythagoras theorem)}$$

$$= 4^2 + 3^2 = 16 + 9 = 25$$

$$BD = 5 \text{ cm}$$

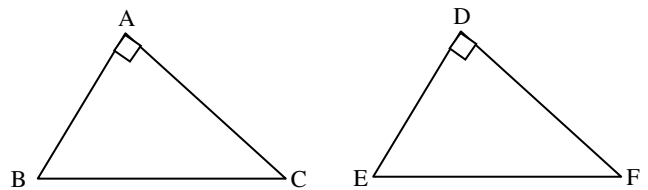
$$\text{In } \triangle ABD, AB^2 = 12^2 = 144.$$

$$BD^2 = 5^2 = 25$$

$$AD^2 = 13^2 = 169 = 144 + 25$$

$$AD^2 = AB^2 + BD^2$$

Hence $\angle ABD = 90^\circ$.



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